In Fig. 1 the results of the calculations are shown of the relative temperature at the boundary of the porous medium in accordance with (20). To compute the temperature of the porous and impenetrable media a program was prepared for the evaluation of the integrals (16)-(17) on the BÉSM-4M electronic computer. In Fig. 2 the results of these calculations are shown verified for the particular case (Fig. 1).

It can be seen from the graphs that the time of reaching steady temperature at the boundary of the porous medium depends strongly on the relation between the thermophysical properties of the porous (with a filler) medium and the impenetrable medium. The highest possible value of $\lambda\sqrt{a}$ should be chosen to reduce the time.

## NOTATION

t, time; x, z, coordinates; $T_1$, $T_2$, $a_1$, $a_2$, $\lambda_1$, $\lambda_2$, temperatures, thermal diffusivities, and thermal conductivities in penetrable and impenetrable half-spaces; u, rate of convective heat transfer by fluid; ε, Joule–Thomson coefficient; $P(\varkappa)$, pressure distribution in porous medium; R, characteristic length; $\eta(x) = \begin{cases} 1, & x > 0; \\ 0, & x < 0. \end{cases}$

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# LOCAL REBINDER CRITERION (NUMBER) OF MOIST DISPERSED SOLIDS

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The results of an analytical and experimental determination of the local Rebinder number of moist dispersed solids are presented. The manner in which the number varies with the properties of the dispersed solids to which it applies is established.

Many experimental and analytical investigations have been applied to the determination of the Rebinder number, a quantity which is employed in calculating rates of drying by the Lykov equation [1] and is defined by the expression

$$\mathrm{Rb} = \frac{c_q}{r} \cdot \frac{d\bar{u}}{d\bar{t}} \tag{1}$$

[1-5]. In all the investigations of which we are aware, however, the Rb number which has been studied has been that characterizing the behavior of the dispersed solid as a whole (the so-called integrated Rb number). In a number of problems relating to the theory of drying it is nevertheless important to know the "local" Rebinder number Rb* relating to an elementary volume of the drying material. A knowledge of the local Rb number is required in the drying of multilayered porous materials and also when calculating heat and mass flows inside the material, which determine the quality of drying (in respect of cracking, shrinkage, local overheating, etc.).

This paper will be devoted to certain properties of the local Rb* number and its relationship to the integrated Rb number.

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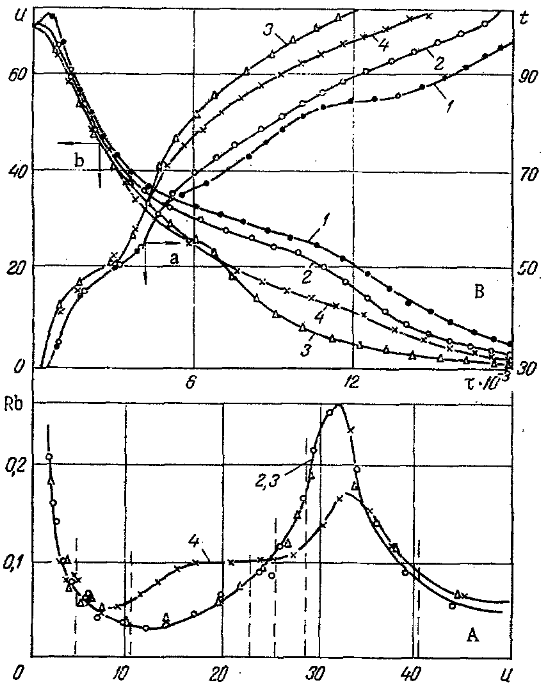

Fig. 1. Relationship between the Rebinder number Rb and the moisture content u (%) (A) and kinetics of the hydrothermal field in silica gel (B) [a) temperature field t, °C; b) moisture-content field u, %; $\tau$, time from onset of drying, sec].

Let us first of all, obtain an analytical expression for Rb*. By definition

$$\mathrm{Rb}^* = \frac{c_q}{r} \cdot \frac{du}{dt}, \tag{2}$$

where u and t represent the average values of the mass content and temperature in the selected elementary volume $\Delta V$.

The differential equations with variable coefficients representing the drying process take the form [1]

$$\frac{\partial t}{\partial \tau} = \nabla (a_q \nabla t) + \frac{\varepsilon r}{c_q} \cdot \frac{\partial u}{\partial \tau}, \tag{3}$$

$$\frac{\partial u}{\partial \tau} = \nabla (a_m \nabla u + a_m \delta \nabla t). \tag{4}$$

The boundary conditions for the selected elementary volume may be written in the form

$$\lambda_q (\nabla t)_s = j_q^* (\tau), \tag{5}$$

$$a_m \gamma_0 (\nabla u)_s + a_m \gamma_0 \delta (\nabla t)_s = -j_m^* (\tau), \tag{6}$$

where $j_q^*(\tau)$ and $j_m^*(\tau)$, respectively, represent the heat and mass flows absorbed in the volume $\Delta V$. We note incidentally that the volume under consideration is regarded as small in the sense that all the characteristics of the material at each point of the volume are the same, but the temperature and mass content at the surface of the volume differ from the values of these quantities inside the volume $\Delta V$, i.e., the elementary volume possesses a perfectly specific heat and mass capacity.

Integrating (3) and (4) within the volume $\Delta V$ and applying the Gauss theorem, we obtain

$$\frac{dt}{d\tau} = \frac{a_q}{R_{\Delta V}} \nabla t + \frac{\varepsilon r}{c_q} \cdot \frac{du}{d\tau}, \tag{7}$$

$$\frac{du}{d\tau} = \frac{a_m}{R_{\Delta V}} \nabla u + \frac{a_m \delta}{R_{\Delta V}} \nabla t, \tag{8}$$

where $R_{\Delta V} = \Delta V / \Delta F$ is the hydraulic radius of the elementary volume, the quantities t and u and the coefficients $\varepsilon$, r, $c_q$ are averaged over the elementary volume $\Delta V$, while the gradients $\nabla t$, $\nabla u$ and the coefficients $a_q$, $a_m$, $\delta$ are averaged over the surface of the volume $\Delta V$.

TABLE 1. Differential Water-Retaining Properties of the Materials Studied

| Material | Adsorbed water | | Maximum amount of water in hygroscopic state, % | State of water in micropores | | Amount of water in case of complete soaking, % |
|---|---|---|---|---|---|---|
| | in monolayer, % | in multilayer, % | | meniscus, % | film-meniscus, % | |
| Silica gel MSM | 5,1 | 10,8 | 23,2 | 25,8 | 29,1 | 40,6 |
| Cellulose | 2,6 | 6,0 | 27,2 | — | — | 50,0 |

In order to determine the average gradients ∇t and ∇u in the boundary conditions (5) and (6), we carry out an integration over the surface of the elementary volume. After substituting the resultant ∇t and ∇u into (7) and (8), we find that

$$\frac{dt}{d\tau}=\frac{1}{c_q\gamma_0 R_{\Delta V}}\overset{*}{j}_q(\tau)-\frac{1}{c_q\gamma_0 R_{\Delta V}}\overset{*}{j}_m(\tau), \tag{9}$$

$$\frac{du}{d\tau}=-\frac{1}{\gamma_0 R_{\Delta V}}\overset{*}{j}_m(\tau), \tag{10}$$

where the flows $j_q^*(\tau)$ and $j_m^*(\tau)$ are averaged over the surface of the volume $\Delta V$. In deriving Eqs. (9) and (10) we assumed (as indicated earlier) that the coefficients averaged over the volume were equal to the corresponding coefficients averaged over the surface of the elementary volume.

Starting from the definition of the local Rb* number (2) and allowing for Eqs. (9) and (10), we obtain

$$\text{Rb}^*=\varepsilon-\frac{\overset{*}{j}_q(\tau)}{r\overset{*}{j}_m(\tau)}. \tag{11}$$

Equation (11) is very similar in form to the basic equation of drying kinetics derived by Lykov [1]:

$$\text{Rb}=1-\frac{j_q(\tau)}{rj_m(\tau)}. \tag{12}$$

A comparison between these equations shows that Rb and Rb* differ from one another by a quantity $(1-\varepsilon)$ for specified rates of flow:

$$\text{Rb}=\text{Rb}^*+(1-\varepsilon). \tag{13}$$

This relationship also corresponds to the physical picture of the process, since in convective drying mass can only be removed from the material as a whole in the form of vapor, whereas the redistribution of moisture between layers may take place in the form of both liquid and vapor; the number $\varepsilon$ takes due account of this.

Furthermore, Eq. (12) may be regarded as a particular case of Eq. (11), since in convective drying (in which all the moisture passes out of the solid in the form of vapor alone) the value of $\varepsilon$ at the surface is equal to 1. Equation (11) thus constitutes the basic equation for the kinetics of local drying in an elementary volume in the interior of the material.

Let us now consider the relationship between the local Rb* and the properties of the material. To this end we use the method of calculation given in [5]. Assuming that one-dimensional heat and mass transfer is taking place, and taking the potential distribution over the thickness of the material in the form

$$t(x)=t_{\text{cen}}-\varphi_q(x)\,(t_{\text{cen}}-t_s), \tag{14}$$

$$u(x)=u_{\text{cen}}-\varphi_m(x)\,(u_{\text{cen}}-u_s), \tag{15}$$

after appropriate transformations we obtain

$$\text{Rb}^*=\varepsilon-\frac{a_q c_q \alpha_q\,[\alpha_m l+K_m^*(l)\,\lambda_m]\,[K_q^*(x)-K_q^*(x+\Delta x)]\,(1+\text{Pn})}{a_m r\alpha_m\,[\alpha_q l+K_q^*(l)\,\lambda_q]\,[K_m^*(x)-K_m^*(x+\Delta x)]}, \tag{16}$$

where

$$K_q^*(x)=\frac{(\nabla\varphi_q)_x\,(t-t_s)}{\frac{1}{l}\int\limits_0^l\varphi_m(x)\,dx-1}, \tag{17}$$

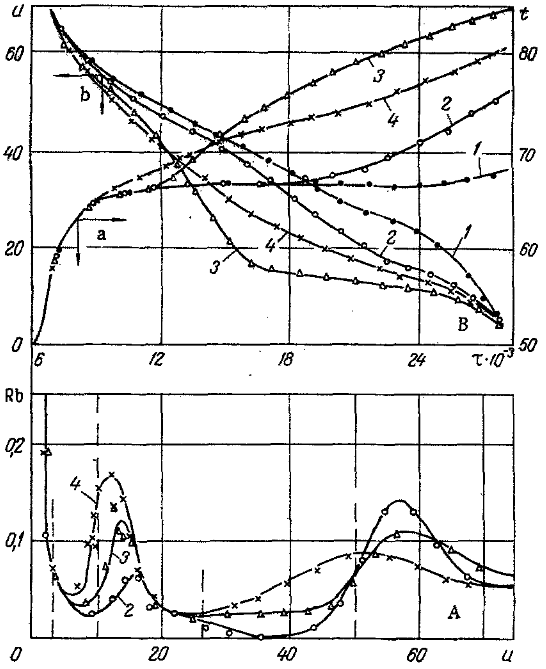

Fig. 2. Relationship between the Rebinder number Rb and the moisture content u (%) (A) and kinetics of the hydrothermal field in cellulose (B) [a) temperature field, t, °C; b) moisture-content field, u, %; $\tau$, time from onset of drying, sec].

$$K_m^*(x) = \frac{(\nabla\varphi_m)_x (u - u_s)}{\frac{1}{l}\int\limits_0^l \varphi_m(x)\,dx - 1}; \tag{18}$$

$\alpha_q$ and $\alpha_m$ are the heat- and mass-transfer coefficients at the surface of the solid, and not that of the selected layer.

Equation (16) relates Rb* to the thermodynamic and transfer characteristics of the material and the specified external drying parameters.

The following expression was derived for Rb in [5]:

$$\mathrm{Rb} = 1 + \frac{a_q c_q \alpha_q K_q(l)\,[a_m \gamma_0 K_m(l)(1 + K_{qm}\mathrm{Pn}) + \alpha_m l]\,(\mathrm{t_{am}} - \bar{\mathrm{t}})}{a_m r \alpha_m K_m(l)(1 + K_{qm}\mathrm{Pn})\,[\alpha_q l + \lambda_q K_q(l)]\,(u_{\mathrm{eq}} - \bar{u})}. \tag{19}$$

Comparison between (16) and (19) enables us to establish a relationship between Rb* and Rb. The first terms in (16) and (19) differ by an amount (1 $-\varepsilon$) as already indicated. The second terms indicate that Rb* and Rb vary in the same way with the thermodynamic and transfer properties of the material and the external heat- and mass-transfer conditions, only differing as regards the coefficients $K_q$ and $K_m$. In general form the distributions of t(x) and u(x) encountered in the course of drying may be quite arbitrary. However, if the form of the distribution functions is the same ($\varphi_q = \varphi_m$), we obtain

$$\mathrm{Rb}^* = \varepsilon - \frac{a_q c_q \alpha_q\,[\alpha_m l + K_m^*(l)\,\lambda_m (1 + \mathrm{Pn})]\,(\mathrm{t_{am}} - \bar{\mathrm{t}})}{a_m r \alpha_m\,[\alpha_q l + K_q^*(l)\,\lambda_q]\,(u_{\mathrm{eq}} - u)}, \tag{20}$$

$$\mathrm{Rb} = 1 - \frac{a_q c_q \alpha_q\,[\alpha_m l + K_m(l)\,\lambda_m (1 + \mathrm{Pn})]\,(\mathrm{t_{am}} - \bar{\mathrm{t}})}{a_m r \alpha_m\,[\alpha_q l + K_q(l)\,\lambda_q]\,(u_{\mathrm{eq}} - \bar{u})}, \tag{21}$$

whence

$$\mathrm{Rb} = \mathrm{Rb}^* + (1 - \varepsilon) \tag{22}$$

for specified external drying conditions ($\alpha_m$, $\alpha_q$, $\mathrm{t_{am}}$, $u_{\mathrm{eq}}$), and not only for specified flows as in (13). It would appear that the distribution functions $\varphi_q$ and $\varphi_m$ are, in fact, very often equal. One example is the parabolic temperature and moisture-content distributions, which have been noted by many authors [6-8].

It follows from the foregoing considerations that the drying of dispersed solids is determined by the two Rebinder numbers: local and integrated; these are not equal to one another, although they are closely related. The number Rb determines the consumption of heat in drying the dispersed solid as a whole, while Rb* enables us to calculate the redistribution of the heat and hence the thermal flows inside the solid (i.e., the kinetic characteristics of the drying of individual parts of the material).

The experimental measurement of Rb and Rb* involves a number of special features. In order to calculate Rb from the drying data it is sufficient simply to know the integrated mass content, which greatly simplifies the experimental technique. In order to determine $Rb^0$, which characterizes the properties of the dispersed material only [5], however, it is essential to work with very large samples under rigorous drying conditions, so that large values of $Bi_q$ and $Bi_m$ are obtained. The resultant values of $Rb^0$ have to be referred to the average $\bar{t}$ and $\bar{u}$ in the material, and these may differ considerably from the t and u of individual layers. This tends to "smooth" the course of $Rb(\bar{t}, \bar{u})$ and impedes analysis of the results.

The number Rb* is free from these disadvantages, but in order to measure it we must know the local mass content, and this greatly complicates the experiments. In addition to this, in order to calculate the properties of the dispersed material on the basis of Rb* by means of Eq. (16) it is essential to know the value of $\varepsilon$, which has to be measured separately. However, $\varepsilon$ may be measured directly by salt or radioactive tracer techniques [9]. Despite the comparatively complicated apparatus requirements, the method of [9] is free from any assumptions as to the transfer mechanism, and hence gives perfectly reliable results. The simultaneous measurement of Rb* and $\varepsilon$ for the same material in principle enables us to calculate the second term in (16), which determines the thermodynamic and transfer properties of the dispersed solid.

In order to illustrate the foregoing, we calculated Rb and Rb* for the convective drying (under symmetrical heat- and mass-transfer conditions) of 30-mm-thick samples of uniformly porous MSM silica gel at $t_{am} = 131.4°C$ and cellulose samples in the form of piles of ashless filter sheets at $t_{am} = 100.5°C$; for this calculation we used experimental curves representing the kinetic characteristics of the temperature and moisture-content fields in silica gel and cellulose in the course of drying.

The local moisture content was measured in several 1-mm-thick layers of the drying sample, using the gammascopic technique [10]. The results of our calculations of the Rebinder numbers are given in Figs. 1A and 2A in the form of Rb(u) curves. Figures 1B and 2B represent the kinetic curves of the temperature and moisture-content fields in the test materials; these enable us to consider the Rb(t) relationship as well. Curves 1-3 in Figs. 1B and 2B illustrate the time change in the temperature (a) and moisture content (b) of the silica gel and cellulose layers, respectively. The disposition of the layers in the materials is identical and corresponds to the following coordinates: 1) center of the plate; 2) 5 mm; 3) 10 mm from the center. Curves 4 reflect changes in the average values of $\bar{t}$ and $\bar{u}$ within the volume of the material. Correspondingly, the local numbers Rb*(u) are illustrated for the corresponding layers by curves 2 and 3, and the integrated numbers Rb(u), by curves 4 in Figs. 1A and 2A. For the central layer the local Rebinder number was not calculated, in view of the fact that the temperature and moisture-content gradients vanished in the center of the plate.

We see from Figs. 1A and 2A that the local and integrated Rb numbers of silica gel and cellulose depend in a complicated manner on the moisture content. This relationship indicates that the form in which the moisture is attached to the materials under consideration exerts a major influence on internal heat and mass transfer. Table 1 gives the moisture content (broken lines in Figs. 1A and 2A) corresponding to the boundaries between the different forms of attachment of the moisture to the silica gel and cellulose samples under consideration, obtained by the drying-thermograph technique [11].

For low moisture contents the values of the integrated and local Rb numbers in the region of adsorbed moisture are identical. There are, nevertheless, considerable differences between these quantities in the region corresponding to the capillary state of the moisture in the micro- and macroscopic pores of the material. The integrated Rb of silica gel (Fig. 1A) exceeds the local Rb* for moisture contents corresponding to the hygroscopic and meniscus states of the water in the pores. In the region of the moist state of silica gel ($u > 25\%$), Rb falls below Rb*. For cellulose (Fig. 2A) the integrated Rb never falls below the local Rb* number, right up to the completely soaked condition. In addition to this, the values of the local Rb* numbers are identical in the two layers of silica gel (Fig. 1A, curves 2 and 3) for equal moisture contents over the whole range of moisture contents, whereas for cellulose (Fig. 2A, curves 2 and 3) they differ in value for $u > 5\%$.

These experimental results agree with the foregoing analytical conclusions. Removal of the adsorbed moisture from the material as a whole and from the elementary layer under consideration takes place in the form of vapor only ($\varepsilon = 1$), and the values of Rb and Rb* thus coincide. The moisture occurring in the capillary state in the micro- and macroscopic pores leaves the drying elementary layer in the form of both liquid and vapor ($\varepsilon < 1$). There should accordingly be a difference of $1 - \varepsilon$ between the values of Rb and Rb*, as already indicated.

In those parts of curves 4 in Figs. 1A and 2A in which $Rb < Rb^*$ it would appear that there is a transient distribution of t and u with respect to the coordinates; this exerts a major influence on the values of the coefficients $K_q$ and $K_m$.

Thus the local Rb number may be regarded as a quantity characterizing the kinetic characteristics of the local drying of an elementary layer in the solid material and also the transfer properties of the latter.

## NOTATION

Rb, Rb*, integral and local Rebinder numbers, respectively; u, $\bar{u}$, local and volume-average moisture contents; t, $\bar{t}$, local and volume-average temperatures; $t_{am}$, ambient (air) temperature; $j_m(\tau)$, $j^*_m(\tau)$, integrated and local moisture flow densities; $j_q(\tau)$, $j^*_q(\tau)$, integrated and local heat flow densities; r, specific heat of vaporization; ε, phase transformation number; $a_q$, $a_m$, heat and moisture diffusion coefficients; $c_q$, specific heat of moist material; $\gamma_0$, density of dry material; $\alpha_q$, $\alpha_m$, heat-and mass-transfer coefficients; δ, thermal-gradient mass-transfer coefficient; $u_{eq}$, equilibrium moisture content of the material; Pn, Posnov number; $\lambda_q$, $\lambda_m$, thermal and mass conductivities of the material; x, coordinate reckoned from the center of the plate; τ, time from the onset of drying. Indices: s, surface of the solid material as a whole or that of the elementary volume; cen, center of the solid.

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# MASS-TRANSFER AND STRUCTURAL-MECHANICS CHARACTERISTICS OF PLASTER OF PARIS

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UDC 66.047

Results are presented on the thermodynamic parameters, mass transfer, porous structure, and state characteristics of plaster of paris.

In recent years, there has been extensive research for new and efficient means of producing modified types of artificial stone on the basis of materials such as plaster of paris, whose parameters are affected by various external factors (temperature, pressure, vibration, magnetic fields, etc.).

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